Study of TE_0 and TM_0 Modes in Dielectric Resonators by a Finite Difference Time-Domain Method Coupled with the Discrete Fourier Transform

A. Navarro, M. J. Nuñez, and E. Martin

Abstract —**We present an application of a numerical method of finite differences in the time domain (FDTD), coupled with the discrete Fourier transform, to determine the resonant frequencies of the TE_0 and TM_0 modes of axially symmetric dielectric resonators closed in a cavity. We analyze the cylindrical cavity dielectrically loaded at the base and the resonant frequency of the $TE_{01\delta}$ mode in two systems: a cylindrical cavity with a cylindrical dielectric resonator of variable radius and the shielded dielectric resonator on a microstrip substrate. The results obtained are compared first with the rigorous (exact) theoretical solutions and then with experimental results.**

I. Introduction

A STUDY of dielectric resonators for application in microwave circuits requires accurate and efficient methods to calculate the resonant frequencies and spatial distribution of the field [1]. Various methods have been developed to study the resonant frequencies of devices in which the dielectric resonator is shielded by one or two conducting planes or is inside a cavity. In the latter case the resonator may be coupled to a microstrip, and usually the existence of a tuning screw is considered. Some of these methods, such as the mode matching method [1]–[5], require some kind of adaptation (for instance, analyzing the mode patterns of the structure) if they are to be applied to different systems. Others, however, such as those of the integral equation formulation [6]–[8] and finite elements [9], [10], are versatile and allow a more general range of applications.

In this paper we present an application of a numerical method [11], based on the coupling of the discrete Fourier transform with the finite difference time-domain (FDTD) method, to the study of TE_0 and TM_0 modes of cylindrical resonators in different enclosures. The method can be used to compute field modes with very general angular dependence [11] and, in comparison with other methods [12], allows quicker and more efficient computation of the global frequency spectrum and distribution of the electromagnetic field.

A simplified version (TE_0 and TM_0 modes) of the method is described in Section II. In Section III we present the results it gives when applied to a structure with a known analytical solution (cavity loaded at the base) and two other structures of practical importance (a cavity loaded with a dielectric resonator and a shielded dielectric resonator on microstrip). The computed results for the resonant frequencies are compared with the experimental results of other authors [13], [14].

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II. Numerical Method

Let us consider, in the absence of sources, a partially homogeneous, axially symmetric closed system (Fig. 1). Let us assume an electromagnetic field which can be expressed as a linear combination of either TE_0 or TM_0 modes. In the cylindrical coordinate system (r,θ,z) associated with the device the field will be determined by the components

$$E_\theta(r,z,t),\quad H_r(r,z,t),\quad H_z(r,z,t)\qquad (TE_0 \text{ modes})$$

or

$$H_\theta(r,z,t),\quad E_r(r,z,t),\quad E_z(r,z,t)\qquad (TM_0 \text{ modes}) \tag{1}$$

without dependence on the angular variable.

Any component of the field (Φ) can be expressed in terms of a superposition of modes $\{\varphi_s(r,z)\}$ with frequencies $\{f_s\}$ as follows:

$$\Phi=\sum_s c_s\cdot\varphi_s(r,z)\cdot e^{j2\pi f_s t},\qquad c_s\in\mathbb{C}. \tag{2}$$

In accordance with Maxwell's equations, in each homogeneous region (of permittivity ϵ and permeability μ) the time evolution of the field will be given by

$$\frac{\partial}{\partial t}\begin{Bmatrix}\epsilon E_\theta\\ -\mu H_\theta\end{Bmatrix}=\frac{\partial}{\partial z}\begin{Bmatrix}H_r\\ E_r\end{Bmatrix}-\frac{\partial}{\partial r}\begin{Bmatrix}H_z\\ E_z\end{Bmatrix}$$
$$\frac{\partial}{\partial t}\begin{Bmatrix}\mu H_r\\ \epsilon E_r\end{Bmatrix}=\frac{\partial}{\partial z}\begin{Bmatrix}E_\theta\\ -H_\theta\end{Bmatrix}$$
$$\frac{\partial}{\partial t}\begin{Bmatrix}\mu H_z\\ \epsilon E_z\end{Bmatrix}=-\frac{1}{r}\cdot\frac{\partial}{\partial r}\left[r\cdot\begin{Bmatrix}E_\theta\\ H_\theta\end{Bmatrix}\right] \tag{3}$$

where the upper quantity is for TE_0 and the lower is for TM_0.

Assume in $t=0$, a field pattern sufficiently abrupt (for instance, zero everywhere except for some points) so as to include, according to (2), a wide group of modes. We now

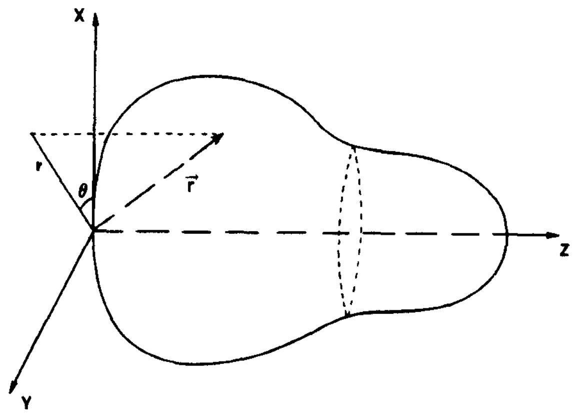

Fig. 1. Geometry and cylindrical coordinates for body of revolution.

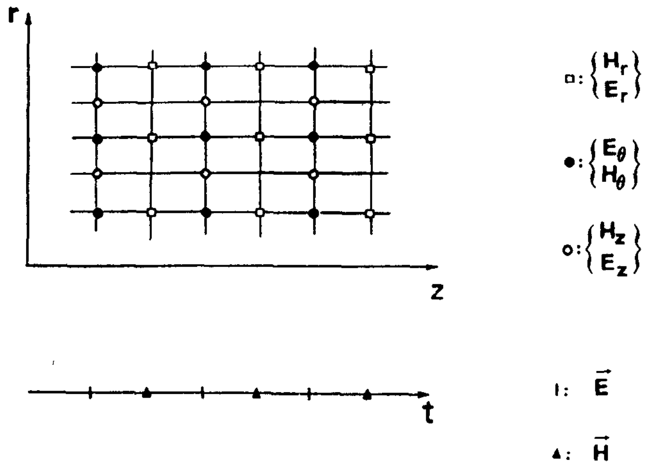

Fig. 2. Space–time mesh used in the calculation of the field components.

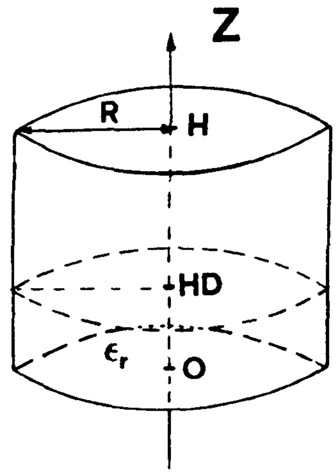

Fig. 3. Cylindrical loaded cavity with radius R and height H, and dielectric at base with thickness HD and relative permittivity ϵ_r.

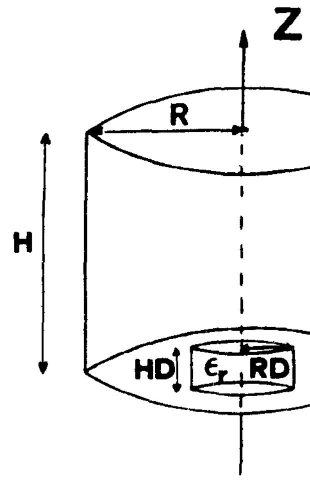

Fig. 4. Dielectric resonator of variable radii RD in a cylindrical cavity. Dimensions are $R = 2.23534$ cm, $H = 3.05764$ cm, and $HD = 0.2025$ cm.

TABLE I
RESONANT FREQUENCIES FOR MODES TE_0 AND TM_0 OF A DIELECTRIC-LOADED CAVITY

Modes	Theoretical Frequency GHz	Numerical Frequency GHz
TM_{011}	5.640	5.64
TE_{011}	8.847	8.84
TM_{021}	9.497	9.486
TM_{012}	11.181	11.17
TE_{021}	12.974	12.96

Temporal discretizations are as follows:

TE_0: $\Delta t = 7.86 \times 10^{-13}$ s; $\Delta f = 0.078$ GHz

TM_0: $\Delta t = 7.07 \times 10^{-13}$ s; $\Delta f = 0.086$ GHz.

consider a space–time mesh covering the whole geometry and an appropriate time interval $(0, T)$; the field components are associated with the mesh nodes in an alternate way, as represented in Fig. 2. From the initial spatial distribution of the field, the imposition in (3) of the boundary conditions of the system (metallic and dielectric boundaries, Z axis of symmetry) enables us to deduce, using a finite-difference algorithm, the field values at successive times, t_i:

$$\Phi_i(r,z) \equiv \Phi(r,z,t_i), \qquad t_i = i \cdot \Delta t$$
$$i = 0, 1, \cdots, N-1, \qquad T = N \cdot \Delta t. \quad (4)$$

The application of the FFT to the time series $\{\Phi_i\}$ associated with a field component at a point (r, z) of the mesh gives the set of coefficients

$$F_k(r,z) = \sum_{i=0}^{N-1} \Phi_i \cdot e^{-j\omega k i}, \qquad \omega = 2\pi / N$$
$$k = 0, 1, \cdots, N/2. \quad (5)$$

The frequency values (f_k) at which $|F_k|$ acquires a local absolute maximum correspond, approximately, to the frequencies $\{f_s\}$ associated with the modes composing the field (eq. (2)). If the frequencies $\{f_s\}$ are sufficiently separated, they can be calculated by the relationship (inferable from the connection between continuous and discrete transform)

$$f_s = \frac{1}{N \cdot \Delta t} \cdot \frac{k \cdot F_k - k' \cdot F_{k'}}{F_k - F_{k'}} \quad (6)$$

where k is the index associated with the maximum of $|F_k|$ corresponding to f_s; k' is one of the neighboring indices (previous or subsequent).

With the frequencies $\{f_s\}$ known, the Fourier coefficients associated with the same frequency value at the different points (r, z) of the mesh give the spatial distribution of the

TABLE II
RESONANT FREQUENCIES FOR $TE_{01\delta}$ MODE OF DEVICE AS IN FIG. 4 WHEN *RD* VARIES ($\Delta t = 7.97 \times 10^{-13}$ s; $\Delta f = 0.0095$ GHz)

RD/R	Measured Frequency GHz	Numerical Frequency GHz	Measured Frequency Shift MHz	Numerical Frequency Shift MHz
1	9.4004	9.3949	0	0
0.9	9.4025	9.3969	2.1	2.0
0.8	9.4132	9.4075	12.8	12.6
0.7	9.4350	9.4285	34.6	33.6
0.6	9.4634	9.4569	63.0	62.0
0.5	9.4921	9.4859	91.7	91.0
0	9.5355	9.5309	135.1	136.0

TABLE III
RESONANT FREQUENCIES FOR $TE_{01\delta}$ MODE OF THE SHIELDED DIELECTRIC RESONATOR IN FIG. 5 ($\Delta t \simeq 3 \times 10^{-13}$ s; $\Delta f \simeq 0.05$ GHz)

RD (mm)	*HD* (mm)	L/HD	L_2/HD	Measured Frequency GH_z	Numerical Frequency GH_z
3.03	4.22	0.943	0.166	8.27	8.25
3.015	3.04	1.69	0.230	9.09	9.04
3.01	2.14	2.83	0.327	10.20	10.12
3.47	2.10	2.90	0.333	8.81	8.78

considered mode:

$$\varphi_s(r,z) \propto F_k(r,z), \qquad k \to f_s.$$

III. APPLICATIONS

The method has been tested by application to structures with a known analytical solution (cylindrical cavity loaded at the base) or structures that have been experimentally studied (dielectric resonator partially loading a cavity [13]; shielded dielectric resonator on microstrip [14]).

We have applied the general scheme, that is, (3)–(6) and the mesh of Fig. 2, to calculate the resonant frequencies of TE_0 and TM_0 modes; they are expressed to the last stable digit when considering various points of the mesh.

Table I presents the frequencies obtained for a cavity (Fig. 3) of radius $R = 1$ cm and height $H = 1.5$ cm loaded by a dielectric with permittivity $\epsilon_r = 10$ and thickness $HD = 0.5$ cm. We have taken a homogeneous space mesh of 46×31 points and a time series of $N = 2^{14}$ times. Table I indicates the value of the time interval, Δt, and frequency $\Delta f\ (=1/T)$ used.

The numerical results are in close agreement with theoretical values. Using (6) instead of directly locating the frequency value from the maximum of $|F_k|$ increases the accuracy while decreasing the error by a factor of approximately 10 (error $\lesssim 1‰$), with the subsequent reductions in CPU time and memory [11].

Subsequently, we applied the method to study the variation in the resonant frequency of the $TE_{01\delta}$ mode for a cavity loaded with a dielectric resonator ($\epsilon_r = 15$) of variable size resting on its base (Fig. 4). For purposes of comparison, we use the experimental results obtained for this device by Krupka [13]. With respect to the geometrical data, we started from the author's values relative to the electrical geometry of the dielectric resonator and we slightly modified the cavity radius and height in order to obtain consistency with the experimental frequencies reported by Krupka in the two situations $R_D/R = 0$ and $R_D/R = 1$.

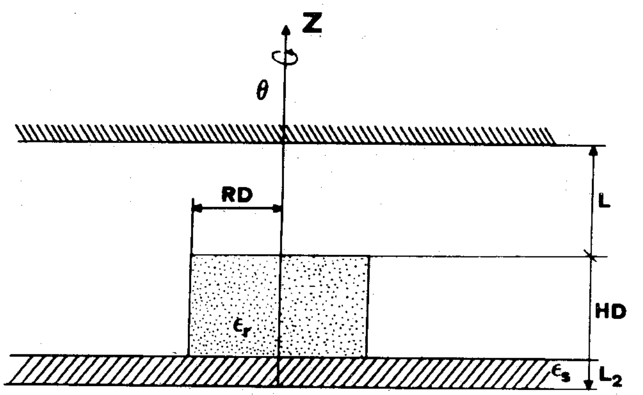

Fig. 5. The shielded dielectric resonator of radius $RD = 4.25$ mm, height $HD = 5$ mm, and dielectric relative permittivity $\epsilon_r = 36.3$. Relative substrate permittivity is $\epsilon_s = 9.25$.

A space mesh (denser in the dielectric region) of 47×31 points was used (see Fig. 2), joined to a time series of $N = 2^{17}$ instants. Table II shows the frequencies obtained as well as their variation according to the situation in which the resonator totally occupied the base. As can be seen, the computed frequency values coincide with theoretical values and those measured within a margin of less than 1‰. This error takes the form of a 5 to 6 MHz frequency offset and it appears to be independent of R_D/R ratio. The systematic character of this deviation results in a close agreement with the values measured when considering a shift in frequency, the discrepancies being lower than 1 MHz.

We have also studied a shielded dielectric resonator coupled to a microstrip (Fig. 5). As FD requires a closed boundary to be applied, we have imposed null field conditions at an appropriate distance from the axis. This can be justified by the high dielectric permittivity of the resonator. The compromise between large distances and a moderate number of mesh points has been achieved with a distance equal to three times the radius of the dielectric resonator.

The method has been applied to the cases experimentally studied in [14]. A spatial mesh of approximately 50×60 nodes was used to generate temporal series of $N = 2^{16}$ time instants. Table III shows the results obtained contrasted with the experimental values. In this case the agreement is within 1%, a reasonable discrepancy bearing in mind the truncation effect and the margin of error in the experimental procedure.

IV. Conclusions

We have described the application of a numerical method, based on coupling the discrete Fourier transform to the finite difference time-domain method to the study of TE_0 and TM_0 modes of cylindrical dielectric resonators. The high precision achieved (discrepancies $\leq 1‰$ for closed devices) is partially due to the algorithm (eq. (6)) used in the frequency calculations; this allows a very important reduction in CPU time and memory. The technique is simple, both conceptually and in its implementation; in addition, it allows access at once to information on the entire modal spectrum by means of the FFT applied to the time series. This results in a technique that offers distinct advantages over other methods used for studying these devices.

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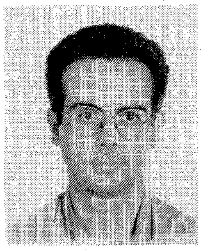

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